MATH 121A Prep: Row Operations

1. Put the matrix $\begin{bmatrix} 0 & 2 & 4 \\ 1 & 1 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ in reduced row echelon form.

Solution: Using the same row operation notation from the video,

$$\begin{bmatrix} 0 & 2 & 4 \\ 1 & 1 & 1 \\ -1 & 1 & 3 \end{bmatrix} \xrightarrow{R1 \leftrightarrow R2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 4 \\ -1 & 1 & 3 \end{bmatrix} \xrightarrow{R3 = R3 + R1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 2 & 4 \end{bmatrix} \xrightarrow{R2 = 1/2 R2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\xrightarrow{R3 = R3 - 2R2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R1 = R1 - R2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

2. Solve the matrix equation $\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & -1 \\ 2 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -1 \\ 0 \end{bmatrix}$

Solution: We will solve the equation by forming an augmented matrix and row reducing.

$$\begin{bmatrix} 1 & -2 & 3 & 10 \\ 0 & -1 & -1 & -1 \\ 2 & 2 & -1 & 0 \end{bmatrix} \xrightarrow{R3 = R3 - 2R1} \begin{bmatrix} 1 & -2 & 3 & 10 \\ 0 & -1 & -1 & -1 \\ 0 & 6 & -7 & -20 \end{bmatrix} \xrightarrow{R2 = -R2} \begin{bmatrix} 1 & -2 & 3 & 10 \\ 0 & 1 & 1 & 1 \\ 0 & 6 & -7 & -20 \end{bmatrix}$$

$$\xrightarrow{R1 = R1 + 2R2} \begin{bmatrix} 1 & 0 & 5 & 12 \\ 0 & 1 & 1 & 1 \\ 0 & 6 & -7 & -20 \end{bmatrix} \xrightarrow{R3 = R3 - 6R2} \begin{bmatrix} 1 & 0 & 5 & 12 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -13 & -26 \end{bmatrix} \xrightarrow{R3 = -1/13 R3} \begin{bmatrix} 1 & 0 & 5 & 12 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\xrightarrow{R2 = R2 - R3} \begin{bmatrix} 1 & 0 & 5 & 12 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R1 = R1 - 5R3} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

So $x_1 = 2$, $x_2 = -1$, $x_3 = 2$ is the solution.

3. One method of finding the inverse of an $n \times n$ matrix A is to form an augmented matrix $[\ A \mid I_n\ ]$ (where I_n is the $n \times n$ identity matrix) and then row reduce until the left hand side becomes the identity. The right hand side is then the inverse, that is after row reduction you have augmented matrix $[\ I_n \mid A^{-1}\ ]$.

   Use this method to find the inverse of $A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -5 & -3 \\ 2 & -3 & 1 \end{bmatrix}$

Solution:

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ -3 & -5 & -3 & 0 & 1 & 0 \\ 2 & -3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R3=R3-2R1]{R2=R2+3R1} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & 1 & 0 \\ 0 & -7 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R3=R3+7R2]{R1=R1-2R2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -5 & -2 & 0 \\ 0 & 1 & 0 & 3 & 1 & 0 \\ 0 & 0 & -1 & 19 & 7 & 1 \end{array}\right] \xrightarrow{R3=-R3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -5 & -2 & 0 \\ 0 & 1 & 0 & 3 & 1 & 0 \\ 0 & 0 & 1 & -19 & -7 & -1 \end{array}\right]$$

$$\xrightarrow{R1=R1-R3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 14 & 5 & 1 \\ 0 & 1 & 0 & 3 & 1 & 0 \\ 0 & 0 & 1 & -19 & -7 & -1 \end{array}\right]$$

Therefore

$$A^{-1} = \begin{bmatrix} 14 & 5 & 1 \\ 3 & 1 & 0 \\ -19 & -7 & -1 \end{bmatrix}$$